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Hydromagnetic Oscillatory Convective Flow through Porous Medium in a Rotating vertical Porous channel with Thermal Radiation Effect

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Abstract

A theoretical analysis of a hydromagnetic oscillatory free convection flow of an electrically conducting viscous incompressible radiating fluid in a vertical porous channel filled with porous medium is analyzed. The parallel porous plates are subjected to a constant injection and suction velocity and a magnetic field of uniform strength is applied perpendicular to the plates of the channel. The magnetic Reynolds number is assumed to be very small so that the induced magnetic field is negligible. The entire system rotates about an axis perpendicular to planes of the plates. The temperature difference between the plates is high enough to induce the heat due to radiation. An exact solution of the purely oscillatory flow is obtained. The velocity and the skin-friction and Nusselt number for the rate of heat transfer in terms of their amplitude and phase angle have been shown graphically to observe the effects of rotation parameter Ω , porosity parameter K , magnetic body force M , Radiation parameter N and the frequency of oscillation ω . Applications of the model include fundamental magneto-fluid dynamics, MHD energy systems and magneto-metallurgical processing for aircraft materials.

Keywords: Darcian regime, radiating fluid, vertical channel, rotating fluid, Hydromagnetic effect, free convection..

Introduction

Flow of a viscous fluid in a rotating medium is of considerable importance due to the occurrence of various natural phenomena and for its application in various technological situations which are governed by the action of Coriolis force. The broad subjects of oceanography, meteorology, atmospheric science and limnology all contain some important and essential features of rotating fluids. The viscous fluid flow problems in rotating medium under different conditions and configurations are investigated by many researchers in the past to analyze various aspects of the problem. The study of simultaneous effects of rotation and magnetic field on the fluid flow problems of a viscous incompressible electrically conducting fluid may find applications in the areas of geophysics, astrophysics and fluid engineering. An order of magnitude analysis shows that, in the basic field equations, the effects of Coriolis force are more significant as compared to that of inertial and viscous forces. Furthermore, it may be noted that Coriolis and magnetohydrodynamic forces are comparable in magnitude and Coriolis force induces secondary flow in the flow-field. Taking into consideration these facts Ghosh and Bhattacharjee [2], Seth and Singh [3], Seth

and Ansari [4], Seth *et al.* [5] and Ghosh *et al.* [6] studied steady MHD flow of a viscous incompressible electrically conducting fluid in a rotating channel under different conditions considering various aspects of the problem. Investigation of oscillatory flow in a rotating channel is important from practical point of view because fluid oscillations may be expected in many MHD devices and natural phenomena where fluid flow is generated due to oscillating pressure gradient or due to vibrating walls. Keeping in view this fact Singh [7], Ghosh [8], Ghosh and Pop [9], Hayat *et al.* [10] and Guria *et al.* [11]. Crammer and Pai [12] investigated oscillatory flow of a viscous incompressible electrically conducting fluid in a rotating channel under different conditions to analyze various aspects of the problem. Rahman and Sattar [13] studied MHD free convection and mass transfer flow with oscillating plate velocity and constant heat source in a rotating frame of reference. On the other hand, Bodoia and Osterle [14], Miyatake *et al.* [15], Lee and Yan [16], Higuera and Ryazantsev [17], Campo *et al.* [18], Pantokratoras [19] have presented their results for a steady free convection flow between vertical parallel plates by considering different

conditions on the wall temperature. Singh *et al.* [20] have studied the transient free convection flow of a viscous incompressible fluid between two vertical parallel plates when the walls are heated asymmetrically. Lee [21] has studied a combined numerical and theoretical investigation of laminar natural convection heat and mass transfer in open vertical parallel plates with unheated entry and unheated exit for various thermal and concentration boundary conditions. Unsteady MHD free convection Couette flow between two vertical parallel plates has been studied by Jha [22]. The study of vertical channel flow bounded by a wavy wall and a vertical flat plate filled with porous medium was presented by Ahmed [23]. Later on Ahmed [24] also investigated the effects of free convection heat transfer on the three-dimensional channel flow through a porous medium with periodic injection velocity. Recently, Fasogbon [25] studied the simultaneous buoyancy force effects of thermal and species diffusion through a vertical irregular channel by using parameter perturbation technique.

Very recently, Sahin and Joaquin [26] obtained the exact solution of the coupled non-linear differential equations governing the heat and mass transfer flow in a rotating vertical channel with Hall currents.

In the present analysis an oscillatory convection flow of an electrically conducting viscous incompressible fluid in a vertical porous channel in a Darcian porous regime is studied. Constant injection and suction is applied at the left and the right infinite porous plates respectively. The entire system rotates about an axis perpendicular to the planes of the plates of the channel and a uniform magnetic field is also applied along this axis of rotation. The magnetic Reynolds number is assumed to be small enough so that the induced magnetic field is neglected.

Basic equations

Under these assumptions we write hydromagnetic governing equations of motion and continuity in a rotating frame of reference as:

$$\nabla \cdot \mathbf{V} = 0, \quad (1)$$

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} + 2\boldsymbol{\Omega} \times \mathbf{V} = -\frac{1}{\rho} \nabla p^* + \nu \nabla^2 \mathbf{V} - \frac{\nu}{K^*} \mathbf{V} + \frac{1}{\rho} (\mathbf{J} \times \mathbf{B}) + \mathbf{F}, \quad (2)$$

$$\rho c_p \left[\frac{\partial T}{\partial t} + (\mathbf{V} \cdot \nabla) T \right] = k \nabla^2 T - \nabla q \quad (3)$$

In equation (2) the last term on the left hand side is the Coriolis force. On the right hand side of (2) the last term $\mathbf{F} (= g\beta T^*)$ accounts for the force due to buoyancy and the second last term is the Lorentz force due to magnetic field \mathbf{B} and is given by

$$\mathbf{J} \times \mathbf{B} = \sigma (\mathbf{V} \times \mathbf{B}) \times \mathbf{B}, \quad (4)$$

and the modified pressure $p^* = p' - \frac{\rho}{2} |\boldsymbol{\Omega} \times \mathbf{R}|^2$, where \mathbf{R} denotes the position vector from the axis of rotation, p' denotes the fluid pressure, \mathbf{J} is the current density, K^* is the permeability of the porous medium and all other quantities have their usual meaning.

Consider the flow of a viscous, incompressible and electrically conducting fluid in a rotating vertical channel immersed in a Darcian porous regime. In order to derive the basic equations for the problem under consideration following assumptions are made:

- The two infinite vertical parallel plates of the channel are permeable and electrically non-conducting.
- The flow considered is fully developed, laminar and oscillatory.
- The fluid is viscous, incompressible and finitely conducting.
- All fluid properties are assumed to be constant except that of the influence of density variation with temperature is considered only in the body force term.
- The pressure gradient in the channel oscillates periodically with time.
- A magnetic field of uniform strength B is applied perpendicular to the plates of the channel.
- The magnetic Reynolds number is taken to be small enough so that the induced magnetic field is neglected.
- Hall effect, electrical and polarization effects are also neglected.
- The temperature of a plate is non-uniform and oscillates periodically with time.
- The temperature difference of the two plates is also assumed to be high enough to induce heat transfer due to radiation.
- The fluid is assumed to be optically thin with relatively low density.
- The entire system (consisting of channel plates and the fluid) rotates about an axis perpendicular to the plates.

Formulation of the problem

In the present analysis we consider an unsteady flow of a viscous incompressible and electrically conducting fluid bounded by two infinite vertical porous plates distance ‘*d*’ apart as shown in Fig. 1. A coordinate system is chosen such that the X^* -axis is oriented upward along the centerline of the channel and Z^* -axis taken perpendicular to the planes of the plates lying in $z^* = \pm \frac{d}{2}$ planes. The fluid is injected through the porous plate at $z^* = -\frac{d}{2}$ with constant velocity w_0 and simultaneous sucked through the other porous plate at $z^* = +\frac{d}{2}$ with the same velocity w_0 . The non-uniform temperature of the plate at $z^* = +\frac{d}{2}$ is assumed to be varying periodically with time. The temperature difference between the plates is high enough to induce the heat due to radiation. The Z^* - axis is considered to be the axis of rotation about which the fluid and the plates are assumed to be rotating as a solid body with a constant angular velocity Ω^* . A transverse magnetic field of uniform strength $\mathbf{B} (0, 0, B_0)$ is also applied along the axis of rotation. All physical quantities depend on z^* and t^* only for this problem of fully developed laminar flow. The equation of continuity $\nabla \cdot \mathbf{V} = 0$ gives on integration $w^* = w_0$. Then the velocity may reasonably be assumed with its components along x^*, y^*, z^* directions as $\mathbf{V} (u^*, v^*, w_0)$.

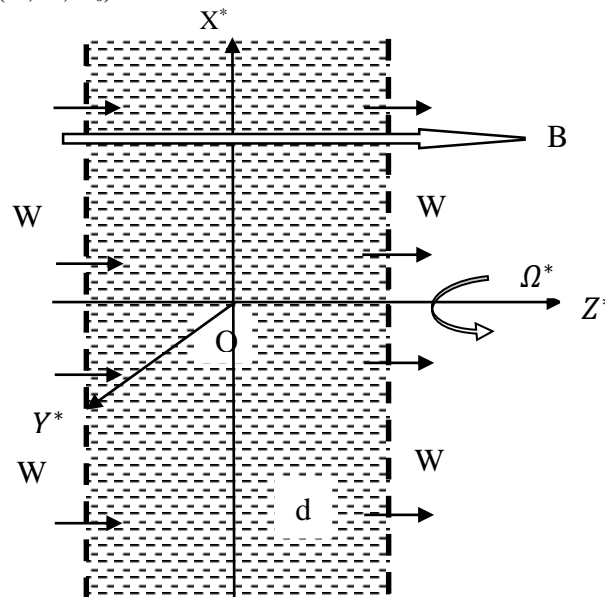


Fig. 1: Coordinate system and the flow configuration

Using the velocity and the magnetic field distribution as stated above the magnetohydrodynamic (MHD) flow in the rotating channel filled with porous medium is governed by the following Cartesian equations:

$$\frac{\partial u^*}{\partial t^*} + w_0 \frac{\partial u^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + \vartheta \frac{\partial^2 u^*}{\partial z^{*2}} + 2\Omega^* v^* - \left(\frac{\sigma B_0^2}{\rho} + \frac{\vartheta}{K^*} \right) u^* + g\beta T^* \tag{5}$$

$$\frac{\partial v^*}{\partial t^*} + w_0 \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \vartheta \frac{\partial^2 v^*}{\partial z^{*2}} - 2\Omega^* u^* - \frac{\vartheta}{K^*} v^* - \frac{\sigma B_0^2}{\rho} v^* \tag{6}$$

$$0 = -\frac{1}{\rho} \frac{\partial p^*}{\partial z^*}, \tag{7}$$

$$\frac{\partial T^*}{\partial t^*} + w_0 \frac{\partial T^*}{\partial z^*} = \frac{k}{\rho c_p} \frac{\partial^2 T^*}{\partial z^{*2}} - \frac{1}{\rho c_p} \frac{\partial q}{\partial z^*} \tag{8}$$

where ‘*’ represents the dimensional physical quantities and the last term in equation (8) is the radiative heat flux. Following Cogley *et al.* [1] it is assumed that the fluid is optically thin with a relatively low density and the heat flux due to radiation in equation (8) is given by

$$\frac{\partial q}{\partial z^*} = 4\alpha^2 T^* \tag{9}$$

where α is the mean radiation absorption coefficient. After the substitution of equation (9) equation (8) becomes

$$\frac{\partial T^*}{\partial t^*} + w_0 \frac{\partial T^*}{\partial z^*} = \frac{k}{\rho c_p} \frac{\partial^2 T^*}{\partial z^{*2}} - \frac{4\alpha^2}{\rho c_p} T^* \tag{10}$$

Equation (7) shows the constancy of the hydrodynamic pressure along the axis of rotation. We shall assume now that the fluid flows under the influence of pressure gradient varying periodically with time in the X*-axis is of the form

$$-\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} = P \cos \omega^* t^* \tag{11}$$

where P is a constant.

The boundary conditions for the problem are

$$z^* = \frac{d}{2}: u^* = v^* = 0, T^* = T_0 \cos \omega^* t^*, \tag{12}$$

$$z^* = -\frac{d}{2}: u^* = v^* = 0, T^* = 0. \tag{13}$$

where T_0 is the mean temperature and ω^* is the frequency of oscillations.

Introducing the following non-dimensional quantities:

$$\eta = \frac{z^*}{d}, x = \frac{x^*}{d}, y = \frac{y^*}{d}, u = \frac{u^*}{w_0}, v = \frac{v^*}{w_0}, T = \frac{T^*}{T_0}, t = \omega^* t^*, p = \frac{p^*}{\rho w_0^2},$$

$$\lambda = \frac{w_0 d}{\vartheta}, \Omega = \frac{\Omega^* d^2}{\vartheta}, M = B_0 d \sqrt{\frac{\sigma}{\mu}}, Gr = \frac{g \beta d^2 T_0}{\vartheta w_0}, Pr = \frac{\mu c_p}{k},$$

$$N = \frac{2 \alpha d}{\sqrt{k}}, \omega = \frac{\omega^* d^2}{\vartheta}, K = \frac{K^* \omega}{d^2}$$

In view of (14), the equations (5), (6) and (10) become

$$\omega \frac{\partial u}{\partial t} + \lambda \frac{\partial u}{\partial \eta} = -\lambda \frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial \eta^2} + 2\Omega v - (M^2 + K^{-1})u + Gr T, \tag{15}$$

$$\omega \frac{\partial v}{\partial t} + \lambda \frac{\partial v}{\partial \eta} = -\lambda \frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial \eta^2} - 2\Omega u - (M^2 + K^{-1})v, \tag{16}$$

$$\omega Pr \frac{\partial T}{\partial t} + \lambda Pr \frac{\partial T}{\partial \eta} = \frac{\partial^2 T}{\partial \eta^2} - N^2 T, \tag{17}$$

The boundary conditions in the dimensionless form become

$$\eta = \frac{1}{2}: u = v = 0, T = \cos t, \tag{18}$$

$$\eta = -\frac{1}{2}: u = v = 0, T = 0. \tag{19}$$

For the oscillatory internal flow we shall assume that the fluid flows under the influence of a non-dimension pressure gradient varying periodically with time in the direction of X-axis only which implies that

$$-\frac{\partial p}{\partial x} = P \cos t \quad \text{and} \quad -\frac{\partial p}{\partial y} = 0. \tag{20}$$

Solution of the problem

Now combining equations (15) and (16) into single equation by introducing a complex function of the form $F = u + iv$ and with the help of equation (20), we get

$$\omega \frac{\partial F}{\partial t} + \lambda \frac{\partial F}{\partial \eta} = \lambda P \cos t + \frac{\partial^2 F}{\partial \eta^2} - (M^2 + K^{-1} + 2i\Omega)F + Gr T, \tag{21}$$

with corresponding boundary conditions as

$$\eta = \frac{1}{2}: F = 0, T = \cos t, \tag{22}$$

$$\eta = -\frac{1}{2}: F = 0, T = 0. \tag{23}$$

In order to solve equation (21) and (17) under boundary conditions (22) and (23) it is convenient to adopt complex notations for the velocity, temperature and the pressure as under:

$$F(\eta, t) = F_0(\eta)e^{it}, \quad T = \theta_0(\eta)e^{it} \quad - \quad \frac{\partial y}{\partial x} = Pe^{it}. \tag{24}$$

The solutions will be obtained in terms of complex notations, the real part of which will have physical significance.

The boundary conditions (22) and (23) in complex notations can also be written as

$$\eta = \frac{1}{2}: F = 0, T = e^{it}, \tag{25}$$

$$\eta = -\frac{1}{2}: F = 0, T = 0. \tag{26}$$

Substituting expressions (24) in equations (21) and (17), we get

$$\frac{d^2 F_0}{d\eta^2} - \lambda \frac{dF_0}{d\eta} - (M^2 + K^{-1} + 2i\Omega + i\omega)F_0 = -\lambda P - Gr \theta_0, \tag{27}$$

$$\frac{d^2 \theta_0}{d\eta^2} - \lambda Pr \frac{d\theta_0}{d\eta} - (N^2 + i\omega Pr)\theta_0 = 0, \tag{28}$$

The transformed boundary conditions reduce to

$$\eta = \frac{1}{2}: F_0 = 0, \theta_0 = 1, \tag{29}$$

$$\eta = -\frac{1}{2}: F_0 = 0, \theta_0 = 0. \tag{30}$$

The solution of the ordinary differential equation (27) under the boundary conditions (29) and (30) gives the velocity field as

$$u(\eta, t) = \left[\frac{1}{2\sinh(\frac{m-n}{2})} \left[\frac{Gr}{2\sinh(\frac{r-s}{2})} \left\{ \left(\frac{e^{\frac{r-s}{2}}}{C_1} - \frac{e^{\frac{r-s}{2}}}{C_2} \right) (e^{m\eta - \frac{n}{2}} - e^{n\eta - \frac{m}{2}}) \right. \right. \right. \right. \tag{31}$$

$$\left. \left. \left. + \left(\frac{C_1 - C_2}{C_1 C_2} \right) (e^{m\eta + \frac{n}{2}} - e^{n\eta + \frac{m}{2}}) e^{-\frac{\lambda Pr}{2}} \right\} + \frac{2\lambda P}{(M^2 + K^{-1} + 2i\Omega + i\omega)} (e^{m\eta} \sinh \frac{n}{2} - e^{n\eta} \sinh \frac{m}{2}) \right] + \frac{\lambda P}{(M^2 + K^{-1} + 2i\Omega + i\omega)} - \frac{Gr}{2\sinh(\frac{r-s}{2})} \left(\frac{e^{r\eta - \frac{s}{2}}}{C_1} - \frac{e^{s\eta - \frac{r}{2}}}{C_2} \right) \right] e^{it},$$

where $C_1 = r^2 - \lambda r - (M^2 + 2i\Omega + i\omega)$, $C_2 = s^2 - \lambda s - (M^2 + 2i\Omega + i\omega)$,
 $m = \frac{\lambda + \sqrt{\lambda^2 + 4(M^2 + K^{-1} + 2i\Omega + i\omega)}}{2}$, $n = \frac{\lambda - \sqrt{\lambda^2 + 4(M^2 + K^{-1} + 2i\Omega + i\omega)}}{2}$,
 $r = \frac{\lambda Pr + \sqrt{\lambda^2 Pr^2 + 4(N^2 + i\omega Pr)}}{2}$, $s = \frac{\lambda Pr - \sqrt{\lambda^2 Pr^2 + 4(N^2 + i\omega Pr)}}{2}$.

Similarly, the solution of equation (28) for the temperature field can be obtained under the boundary conditions (29) and (30) as

$$T(\eta, t) = \left(\frac{e^{r\eta - \frac{s}{2}} - e^{s\eta - \frac{r}{2}}}{2\sinh(\frac{r-s}{2})} \right) e^{it}. \tag{32}$$

From the velocity field obtained in equation (31) we can get the skin-friction τ_L at the left plate ($\eta = -0.5$) in terms of its amplitude $|F|$ and phase angle φ as

$$\tau_L = |F| \cos(t + \varphi), \text{ with} \tag{33}$$

$$F = F_r + i F_i = \frac{1}{2\sinh(\frac{m-n}{2})} \left[\frac{Gr}{2\sinh(\frac{r-s}{2})} \left\{ \left(\frac{e^{\frac{r-s}{2}}}{C_1} - \frac{e^{\frac{r-s}{2}}}{C_2} \right) (m - n) e^{-\frac{\lambda}{2}} + \left(\frac{C_1 - C_2}{C_1 C_2} \right) (m e^{-\frac{m-n}{2}} - n e^{-\frac{m-n}{2}}) e^{-\frac{\lambda Pr}{2}} \right\} + \frac{2\lambda P}{(M^2 + K^{-1} + 2i\Omega + i\omega)} (m e^{-\frac{m}{2}} \sinh \frac{n}{2} - n e^{-\frac{n}{2}} \sinh \frac{m}{2}) \right] - \frac{Gr}{2\sinh(\frac{r-s}{2})} \left(\frac{r}{C_1} - \frac{s}{C_2} \right) e^{-\frac{\lambda Pr}{2}}. \tag{34}$$

The amplitude is $|F| = \sqrt{F_r^2 + F_i^2}$ and the phase angle is $\varphi = \tan^{-1} \frac{F_i}{F_r}$. (35) Similarly the Nusselt number Nu in terms of its amplitude $|H|$ and the phase angle ψ can be obtained from equation (32) for the temperature field as

$$q = |H| \cos(t + \psi), \tag{36}$$

$$\text{with } H = H_r + i H_i = \frac{(r - s)e^{-\frac{r+s}{2}}}{2 \sinh(\frac{r-s}{2})}, \tag{37}$$

where the amplitude $|H|$ and the phase angle ψ of the rate of heat transfer are given as

$$|H| = \sqrt{H_r^2 + H_i^2}, \quad \psi = \tan^{-1} \left(\frac{H_i}{H_r} \right). \tag{38}$$

Results and discussion

The study of oscillatory magnetohydrodynamic convective and radiative MHD flow in a vertical porous channel in a Darcian porous regime is analyzed. The fluid is injected through one of the porous plates and simultaneously

removed through the other porous plate with the same velocity. The entire system (consisting of Porous channel plates and the fluid) rotates about an axis perpendicular to the plates. The closed form solutions for the velocity and temperature fields are obtained analytically and then evaluated numerically for different values of parameters appeared in the equations. To have better insight of the physical problem the variations of the velocity, temperature, skin-friction rate of heat transfer in terms of their amplitudes and phase angles are evaluated numerically for different sets of the values of physical parameters.

The variations of the velocity profiles with the magnetic parameter M are presented in Fig. 2. As the Lorentz force acts opposite to the flow motion and therefore it slow down the motion and hence the magnetic field retards the flow velocity. Also the maximum of the velocity profiles shifts toward right half of the channel due to the rotation of the entire system. Fig.3 illustrates the variation of the velocity with the increasing rotation of the system. It is quite obvious from this figure that velocity goes on decreasing with increasing rotation Ω of the entire system. The maximum velocity has occurred near the right plate ($\eta=0.5$) and the minimum velocity near the left plate ($\eta=-0.5$) for all values of rotation parameter Ω . The variation of the velocity profiles with the porosity K is presented in Fig.4. The velocity goes on increasing with increasing K and remains parabolic with maximum at the centerline. The effects of rotation and magnetic field on the amplitude $|F|$ of skin-friction τ_L at the left plate $\eta=-0.5$ has been shown in Figs. 5. It depicts that the skin-friction amplitude decreases with the increase of Hartmann number M , but significantly $|F|$ has no effects for the large values of $M=10, 15$. Also it has been seen that, $|F|$ increases near the plate for different values of Ω , but away the plate it is looking insignificant. The effects of rotation and magnetic field on the phase angle (ϕ) of skin-friction τ_L at the left plate $\eta=-0.5$ are exhibited in Fig.6. It is noticed that phase angle decreases with increasing magnetic field M . However, phase angle has risen for different values of Ω . The phase angle (ψ) of the rate of heat transfer for different values of frequency of oscillations (ω) and thermal radiation (N) are illustrated in Fig. 7. It is evident that the phase angle (ψ) decreases with the increase of radiation parameter. Significantly the frequency of oscillations boosts the phase angle (ψ). Table 1 shows the distribution of amplitude of rate of heat transfer, $|H|$ for different values of frequency of oscillations (ω) and thermal radiation (N). It has been seen that the amplitude is depressed for the effect of N , but $|H|$ is elevated for increasing values of ω .

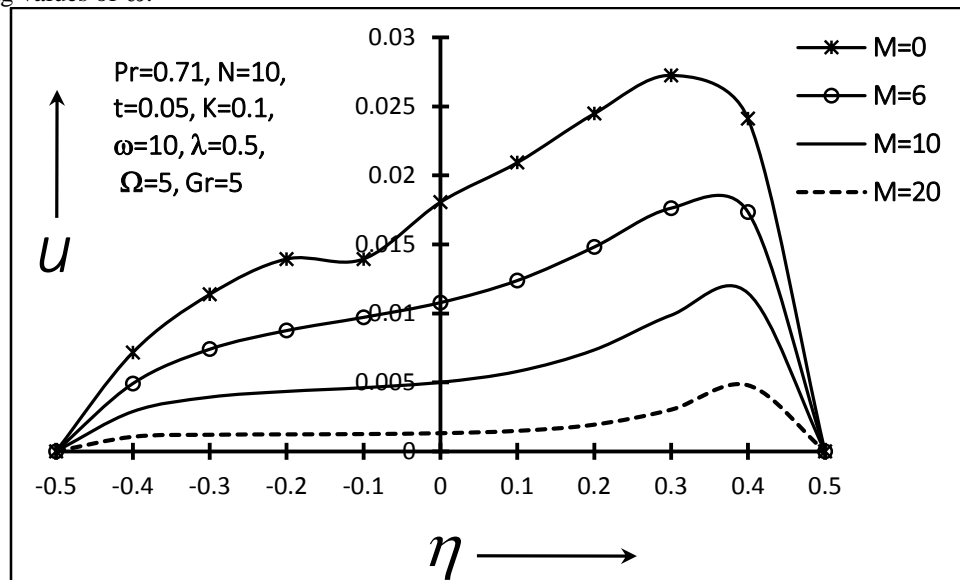


Fig. 2: Velocity profile for magnetic field

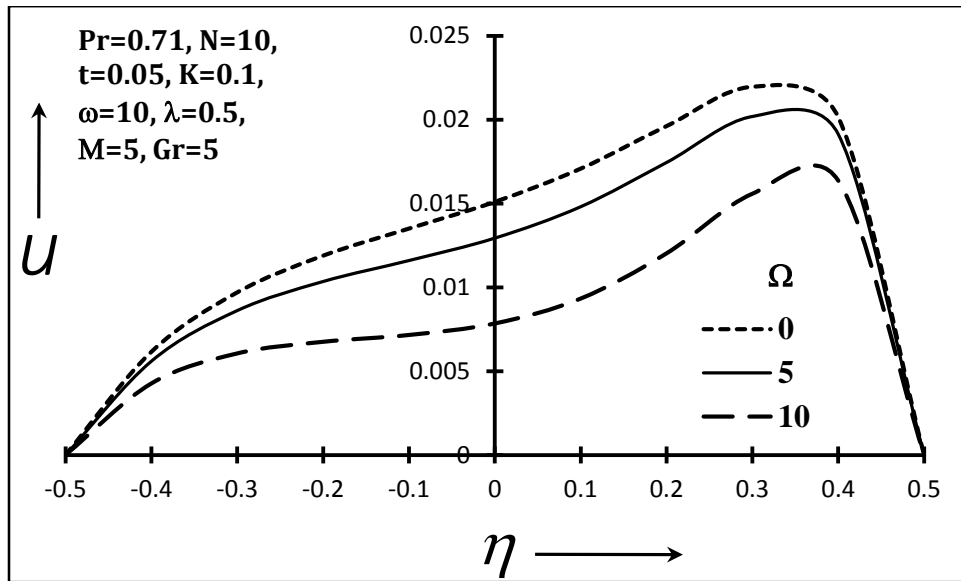


Fig. 3: Velocity profile for rotation parameter

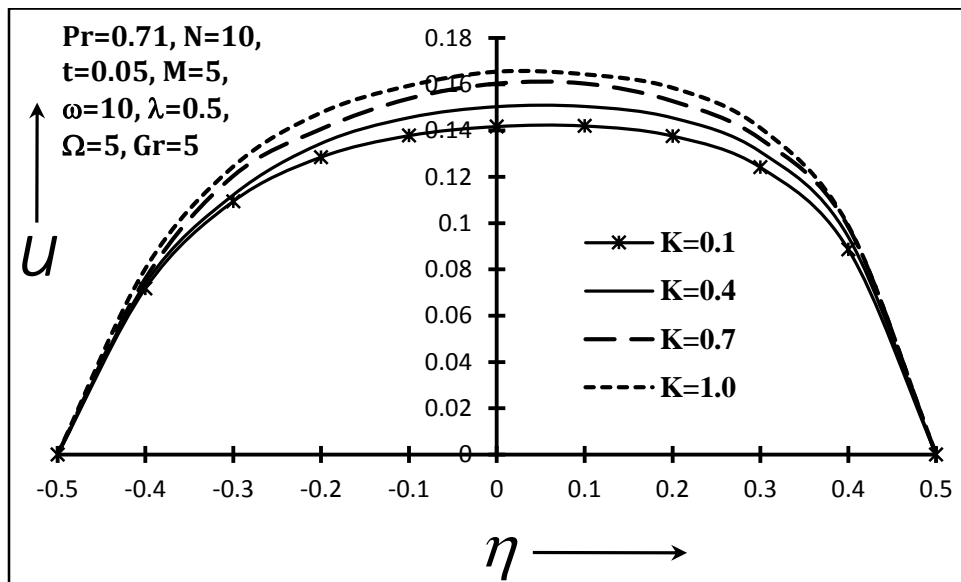


Fig. 4: Velocity profile for porosity

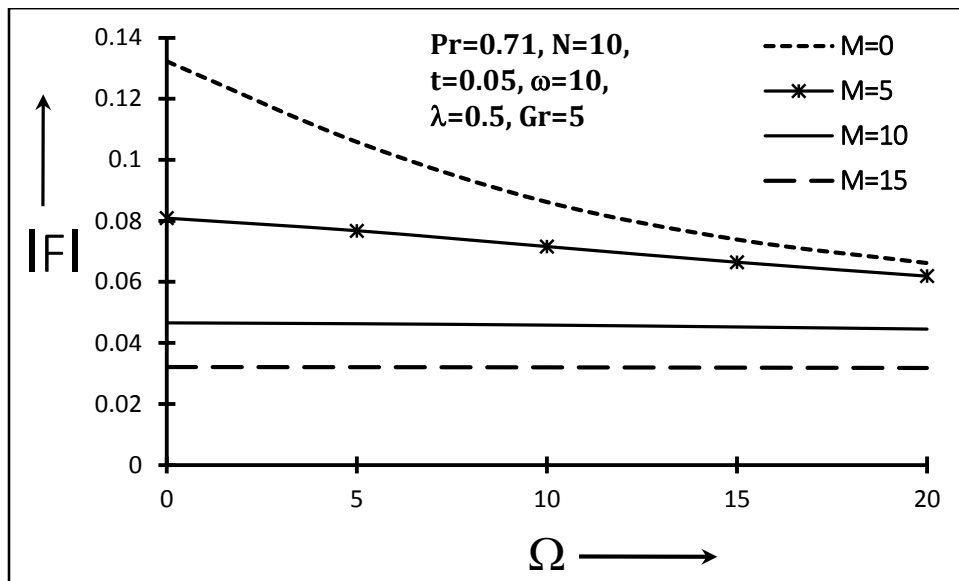


Fig. 5: Amplitude of skin-friction for M and Ω

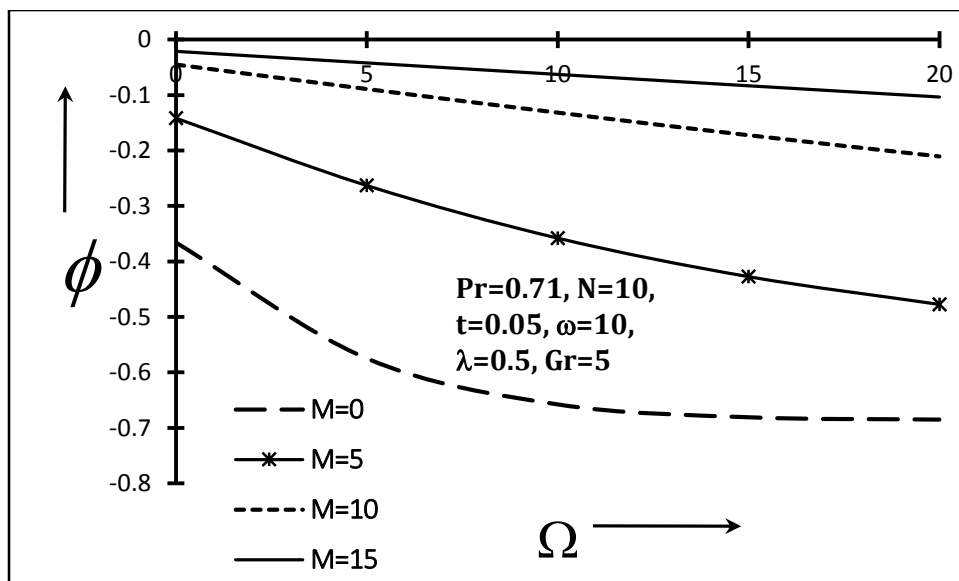


Fig. 6: Phase angle of skin-friction for M and Ω

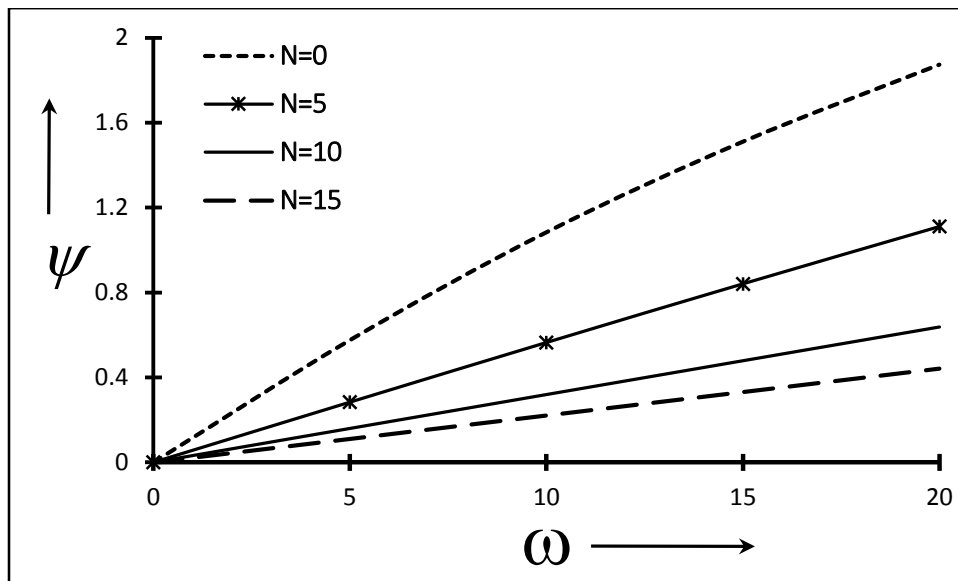


Fig. 7: Phase angle of heat transfer for N and Ω

Table-1: Amplitude of heat transfer for the effects of radiation and frequency of oscillation

ω	$N=0$	$N=5$	$N=10$	$N=15$
0	0	0	0	0
5	0.57588	0.28348	0.15975	0.11043
10	1.08374	0.56452	0.31929	0.22084
15	1.51058	0.84087	0.47863	0.33192
20	1.87338	1.11083	0.63761	0.44155

Conclusions

Unsteady hydromagnetic convective flow of a viscous incompressible electrically conducting fluid within a parallel plate channel in a porous medium is investigated when the entire system consisting of channel plates and the fluid rotates about an axis perpendicular to the plates. A closed form solution of the problem is obtained. The significant findings are summarized below:

- The magnetic body force is decelerating the flow velocity;
- The rotation of the system is depressed the flow velocity;
- The porosity of the medium is increased the flow velocity;
- Maximum rotation of the system has been observed on the flow velocity at the right plate;
- The magnetic body force/thermal radiation reduced the phase angle of skin friction;

- The rotation/frequency of oscillation increased the phase angle of skin friction/rate of heat transfer;
- The increase of thermal radiation reduces the amplitude of rate of heat transfer.

Nomenclature

- B_0 component of the applied magnetic field along the z^* -axis
- C_p specific heat at constant pressure
- g gravitational force
- Gr Grashoff number
- κ Thermal conductivity
- M Hartmann number
- N Heat radiation parameter
- P a constant
- p pressure
- Pr Prandtl number
- t time variable
- T fluid temperature
- T_0 constant temperature

u, v, w	velocity components along X, Y, Z-directions
w_0	injection/suction velocity
x, y, z	variables along X, Y, Z-directions
λ	Injection/suction parameter,
K	Porosity parameter,
α	Mean radiation absorption coefficient
β	Coefficient of volume expansion
ω	Frequency of oscillations
ϑ	Kinematic viscosity
ρ	fluid density
σ	Electric conductivity
Ω	rotation parameter
τ_L	skin-friction at the left wall
φ	phase angle of the skin-friction
θ_0	mean non-dimensional temperature
*	superscript representing dimensional quantities

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